Conceptual Foundations of Quantum Field Theory

I shall concentrate on the problem of wave-particle duality:

Wave and particle descriptions apply in complementary contexts

But Dirac (1927) says (& FT means waves or particles in all situations

"Complete harmony between the wave and light-quantum description"

What did Dirac mean and was he right?

This raises the issue
of <u>underdetermination</u>

How should we respond to
the empirical equivalence
of two metaphysically distinct
descriptions?

Arrogance: If we cannot
know something, there
is no fact of the matter
for us to be ignorant
about.

Ignorance: We don't know
everything!

Role of heuristics in resolving
underdetermination

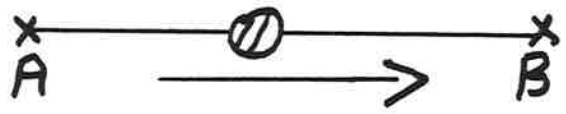
CLASSICAL CONCEPT OF FIELD

FIELD THEORY: Associates
Contain properties with
Space-Time Points

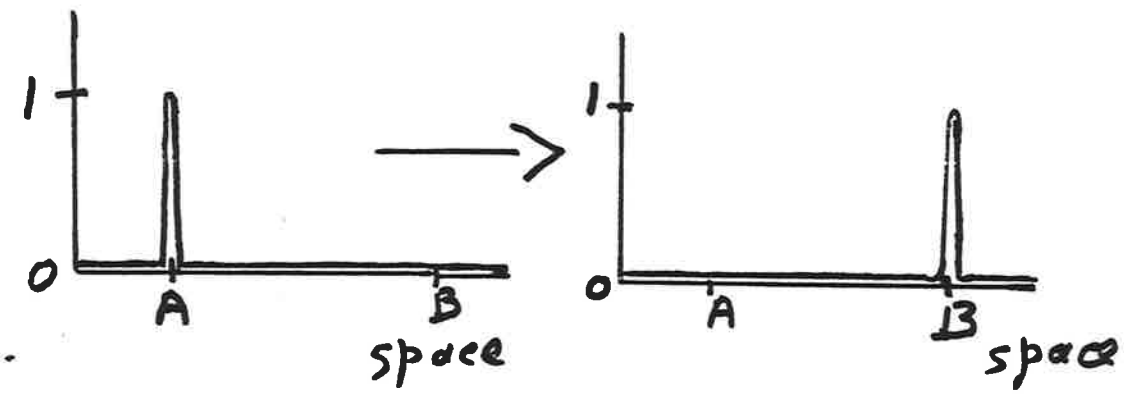
Compare
PARTICLE THEORY: Attributes
To contain individuals
(the particles) a variety
of properties

MOTION FROM A TO B

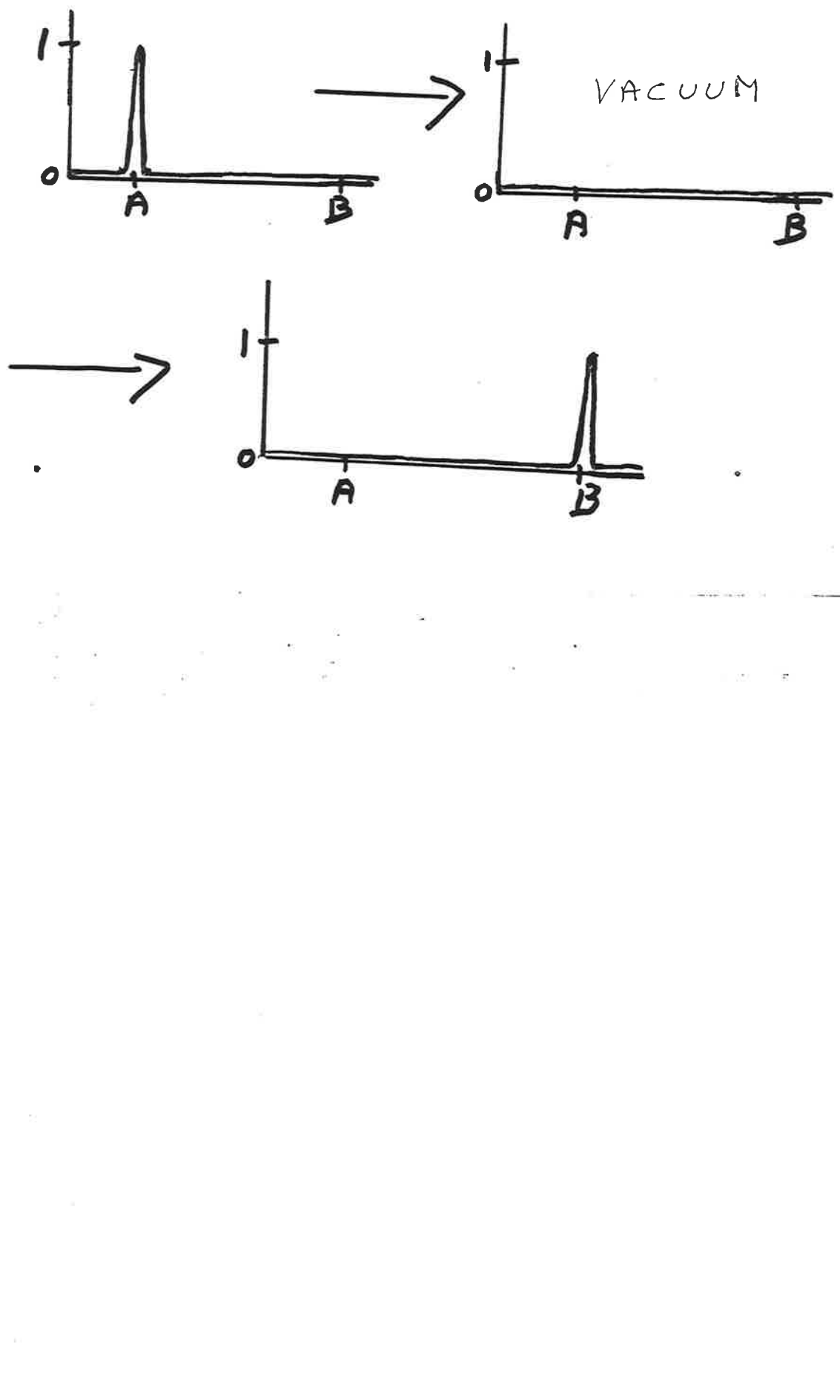
Particle Description



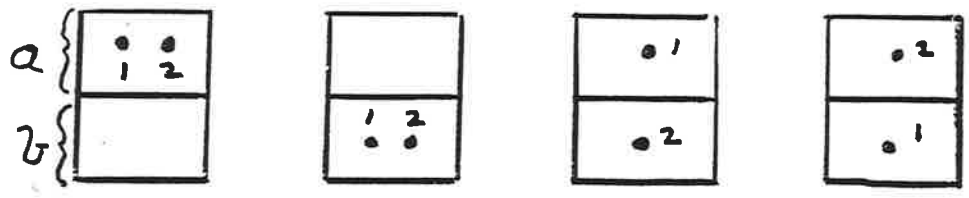
Field Description



ANNIHILATION CREATION AND THE VACUUM



Statistical weights for 2-particle System



In Quantum Stat. Mech. (III) and (IV) are regarded as one and the same state for the purposes of assigning Statistical weights

Quantum Statistical Mechanics

Consider the 4 product wave functions

$$\psi_a(\underline{r}_1)\cdot\psi_a(\underline{r}_2) \quad \text{I}$$
$$\psi_b(\underline{r}_1)\cdot\psi_b(\underline{r}_2 \quad \text{II}$$
$$\psi_a(\underline{r}_1)\cdot\psi_b(\underline{r}_2) \quad \text{III}$$
$$\psi_a(\underline{r}_2)\cdot\psi_b(\underline{r}_1) \quad \text{IV}$$

4-dimensional vector space equally well spanned by

Symmetric
$$\psi_a(\underline{r}_1)\cdot\psi_a(\underline{r}_2) \quad \text{V}$$
$$\psi_b(\underline{r}_1)\cdot\psi_b(\underline{r}_2) \quad \text{VI}$$
$$\frac{1}{\sqrt{2}}\left(\psi_a(\underline{r}_1)\cdot\psi_b(\underline{r}_2)+\psi_a(\underline{r}_2)\cdot\psi_b(\underline{r}_1)\right) \quad \text{VII}$$

Antisymmetric
$$1/\sqrt{2}\left(\psi_a(\underline{r}_1)\cdot\psi_b(\underline{r}_2)-\psi_a(\underline{r}_2)\cdot\psi_b(\underline{r}_1)\right) \quad \text{VIII}$$

Even in NR QFT arguments against equivalence thesis arise

For an infinite number of degrees of freedom Hilbert space becomes nonseperable — inequivalent representations of algebra of commutation relations

In particular non-Fock representations based on states with an 'infinite' number of particles, exist.

In RQFT this problem persists and manifests itself in Haag's Theorem (1955).

Query: Is Haag's Theorem devasting for RQFT?

Not really, because in LSZ formulation of scattering theory we only assume weak convergence of interacting field to asymptotic free field — objects like the Møller wave matrix are not assumed to exist (and do not by Haag's theorem)

In RQFT additional problems arise.

Two concepts of position
- one for particles and
  one for fields

Historical background

For Dirac eq. problem of defining a velocity operator
- appears to be $2c$, with eigenvalues $\pm c$!

Zitterbewegung – Schrödinger (1930)

Foldy – Wouthuysen Transformation (1950)

↳ extended to spin zero particles by Case (1954)
cf Feshbach & Villars Review (1958)

General definition of position operator for a relativistic theory given by Newton - Wigner (1949) (does not exist for the photon, so is the photon not a particle?)

More general discussion by Wightman (1962) using Mackey's idea of a system of imprimitivity

— closely related to Mackey's work on induced representations for locally compact Lie groups with a semi-direct product structure, like the Poincaré group.

In 1964 Weinberg showed how to relate infinite-dimensional unitary representations of the Poincaré group to the finite-dimensional non-unitary reps of the Lorentz group, using his method of auxiliary groups to factorize the representation of the Wigner rotations — this was in effect the inverse of the F-W transformation (Cp Fonda and Ghirardi (1968))

Finally compare Segal approach (Geometric quantization)

— two sorts of complex structure in going from classical symplectic manifold to 'Complex coordinates', and then doing RBFT in the space of Analytic functions of these coordinates.

— This work is again deeply related to the theory of Unitary reps of non compact groups due to Kostant, Souriau, Kirillov etc.

Brief Exposition of the N-W states for Klein-Gordon equation

On positive energy sector of the mass hyperboloid

$$\psi(\underset{\sim}{x}) = \frac{1}{\sqrt{(2\pi)^3}} \int \frac{d^3\underset{\sim}{k}}{\omega(\underset{\sim}{k})} a(\underset{\sim}{k}) e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}} \quad (\text{at } t=0)$$

with inner product

$$\langle \psi_1 | \psi_2 \rangle = \int \frac{d^3\underset{\sim}{k}}{\omega(\underset{\sim}{k})} a_1^*(\underset{\sim}{k}) a_2(\underset{\sim}{k})$$

Some choices for $a(\underset{\sim}{k})$

$$a(\underset{\sim}{k}) = \omega(\underset{\sim}{k})\,\delta(\underset{\sim}{k}-\underset{\sim}{k}')$$

$$\Rightarrow \psi_{\underset{\sim}{k}'}(\underset{\sim}{x}) = \frac{1}{\sqrt{(2\pi)^3}} e^{i\underset{\sim}{k}'\cdot\underset{\sim}{x}}$$

$$a(\underset{\sim}{k}) = \frac{1}{\sqrt{(2\pi)^3}}\,\omega(\underset{\sim}{k})\, e^{-i\underset{\sim}{k}\cdot\underset{\sim}{z}}$$

$$\Rightarrow \phi_{\underset{\sim}{z}}(\underset{\sim}{x}) = \delta(\underset{\sim}{z}-\underset{\sim}{x})$$

$$a(\underset{\sim}{k}) = \frac{1}{\sqrt{(2\pi)^3}} \sqrt{\omega(\underset{\sim}{k})}\, e^{-i\underset{\sim}{k}\cdot\underset{\sim}{z}}$$

$$\Rightarrow \psi_{\underset{\sim}{z}}(\underset{\sim}{x}) \sim H^{(1)}_{5/4}\left(|\underset{\sim}{z}-\underset{\sim}{x}|\right) \sim |\underset{\sim}{z}-\underset{\sim}{x}|$$

Note $\langle \phi_{\underset{\sim}{3}}' | \phi_{\underset{\sim}{3}} \rangle \neq 0$

But $\langle \psi_{\underset{\sim}{3}}' | \psi_{\underset{\sim}{3}} \rangle = \delta(\underset{\sim}{3}' - \underset{\sim}{3})$

we can introduce number operators

$$N(x) = N^+(x) + N^-(x)$$

$$N(\underset{\sim}{3}) = N^+(\underset{\sim}{3}) + N^-(\underset{\sim}{3})$$

also charge density

$$Q(x) = e(N^+(x) - N^-(x)) + \text{fluctuation terms}$$

$Q(\underset{\sim}{x})$ and $N(\underset{\sim}{3})$ are local but $N(\underline{x})$ is not.

Note that $Q(x)$, unlike $N(x)$ can be extended in terms of a causal field

$$\psi(x) = \frac{1}{\sqrt{2}}(\psi^+(\underset{\sim}{x}) + \psi^-(\underset{\sim}{x}))$$

Causality $\Rightarrow$ spin-statistics theorem + PCT theorem

Query: What do detectors detect?

— particles or charge

cf Hegenfeldt's Theorem (1974)
Can it be tested experimentally?

See Ruijsenaars
Annals of Physics
137, 33 (1981)

Also paradoxes of relativistic measurement theory
— Bloch (1967)
Aharonov and Albert (1980)

Resolution using Fleming's
F-dimensional hyperplane-dependence theory

Relativistic Particles behave in a sense like extended objects.

— cp non local field theories → string theory.

All this is for free fields on Minkowski space-time
What about interacting fields — virtual particles, distinction between matter and force etc.?
ds in curved space-time the ambiguities in the notion of particle associated with Rindler quanta and the Unruh effect

Conclusion

In RQFT, the notion of particle is ambiguous, and far-removed from the classical notion of an entity whose localization, indeed very existence, may be relativized to the observer.

But should we just be more relaxed in what we mean by particle?

In NRQFT, there is formal underdetermination, but this does not solve the problem of wave-particle duality in the way Dirac intended. There is still plenty of complementarity, e.g. between particle number and wave phase